Tables of properties: examples and constructions

Topology FS 2019

Francesco Fournier Facio

July 1, 2019

These two tables concern the behaviour of topological properties you have encountered this semester under certain constructions. The first table contains many examples, that should serve as a basis to complete the second table, containing the constructions. You are encouraged to write down the empty table and try to complete it yourself, coming to this document when you need help: it is an excellent way to prepare for the exam. Here is the list of properties, examples and constructions that appear in the table.

Properties.

- Compact.
- Locally compact.
- Connected.
- Path-connected.
- Locally path-connected.
- First-countable.
- Second-countable. (*Remark.* Whenever we consider the properties of being first- or second-countable, we will always assume that we are working in uncountable spaces. This is because, although there are countable spaces that are not first- or second-countable (notice that for a countable space, these properties are equivalent), the examples are quite technical and I didn't want to get into that.)
- Hausdorff (T2).
- Regular (T3).
- Normal (T4).
- Metrizable.
- Complete.
- Simply connected (*Remark.* In this document simply connected means path-connected and trivial fundamental group: some people don't require path-connectedness).

- Contractible.

Examples

- An infinite set X with the trivial topology $\mathcal{T}_{tr}$.
- An infinite set X with the discrete topology $\mathcal{T}_{disc}$.
- An infinite set X with the cofinite topology $\mathcal{T}_{cof}$ (defined in Ex. 5.1).
- An infinite set X with the particular point topology $\mathcal{T}_p$, where $p \in X$. This is the topology where a non-empty set is open if and only if it contains p.
- The real line $\mathbb{R}$ with the standard topology.
- The circle S^1 with the standard topology.
- The sphere S^n, for $n > 1$, with the standard topology.
- The product $\mathbb{R}^\omega$ of countably many copies of $\mathbb{R}$ with the product topology $\mathcal{T}_{prod}$.
- The product $\mathbb{R}^\omega$ of countably many copies of $\mathbb{R}$ with the box topology $\mathcal{T}_{box}$.

Constructions

- Subspace.
- Product (finite or infinite).
- Quotient.
- Image under a continuous map.
- Homeomorphism.
- Closure (i.e., if $A \subseteq X$ and A has a given property with its subspace topology, does $\overline{A}$ also have that property with its subspace topology?).
- Finer topology (i.e., more open sets).
- Coarser topology (i.e., less open sets).

There are also other properties you might want to look at that I didn't mention here. Sequential compactness comes to mind for example. Also for the constructions: you might want to look at image under a continuous open map, or at the one-point compactification.

1 How to read the tables

You should start by filling in the table of examples, since it gives counterexample to many constructions. For instance, the reason the particular point topology is here, is that it has the property that the closure of the particular point p is the whole space. So if you have a property that is not satisfied by $(X, \mathcal{T}_p)$, but it is satisfied by the space with a single point, then it is not stable under closure.

Another great source of counterexamples is the inclusion of topologies $\mathcal{T}_{tr} \subset \mathcal{T}_{cof} \subset \mathcal{T}_{disc}$ on any set. If you choose a space with its own standard topology, then that topology will probably sit somewhere in this inclusion (in the case of $\mathbb{R}$, between $\mathcal{T}_{cof}$ and $\mathcal{T}_{disc}$). This gives counterexamples to the columns about finer/coarser topologies. Another source of counterexamples for this is the inclusion of topologies $\mathcal{T}_{prod} \subset \mathcal{T}_{box}$ on $\mathbb{R}^\omega$.

Here is how to read the table. A **green tick** means that the property is satisfied for the example/stable under this construction, a **red cross** means it is not. When instead of a tick or a cross you find a **word**, it means that this works under this additional condition, and it does not otherwise. For instance, some properties may be stable under finite products, but not under arbitrary products.

If an entry contains nothing else (meaning no red star, no green circle and no blue square), then it means that it has been done in the class, is in the exercises, or is a direct consequence of other informations you have already from the table.

With regard to the other symbols, here is their meaning. A **red star** next to the entry means that the proof or counterexample goes beyond what you are supposed to know or be able to do at the end of this class, and therefore is not going to be asked at the exam. A **green circle** means that you are not supposed to know this already, but it is an exercise. A **blue square** means that it is a hard exercise, those are the ones that we will not ask at the exam, unless it is already going very very well :) .

There are a few special cases. In some cases you find both a green circle and a star next to an entry with an additional condition: it means that is doable to prove that this additional condition works, but the counterexample for the general case is hard; or vice-versa! Finally, for whether $(\mathbb{R}^\omega, \mathcal{T}_{box})$ is normal, you can ask Prof. Sisto for a hint...

... just kidding: it's still an open problem.

In what follows you find solutions to the green circles and blue squares and references for the red stars. The solutions are sometimes only sketches. The references will consist of hyperlinks or to references to Munkres (second edition).

2 Examples table

2.1 Green circles

$(X, \mathcal{T}_{tr})$ **is contractible**.
We need to show that there exists a continuous map $H : X \times I \to X$ and a point $p \in X$ such that for all $x \in X$ we have $H(x, 0) = x$ and $H(x, 1) = p$. But any map from a topological space to a space with the trivial topology is continuous, so we can define H arbitrarily on $X \times (0, 1)$, and it will give us the desired contraction.

This also implies that it is **path-connected** (although you could use the same argument and show that any map $I \to X$ is continuous) and **simply connected**. Finally, the unique non-empty open set provides a basis of path-connected open sets for the topology, so X is also **locally path-connected.**

$(\mathbb{R}^\omega, \mathcal{T}_{prod})$ **is not locally compact**.
Let $\mathbf{x} \in \mathbb{R}^\omega$, and pick a neighbourhood U of $\mathbf{x}$. We want to show that U cannot be contained in a compact set. Indeed, by definition of the product topology, U must contain a basic open set V, which is a product of open sets in $\mathbb{R}$, only finitely many of which are not the whole of $\mathbb{R}$. Let $N \geq 1$ be such that $\pi_N(V) = \mathbb{R}$. Then for any K containing U, we have $\pi_N(K) = \mathbb{R}$ as well. But π_N is continuous and $\mathbb{R}$ is not compact, so K cannot be compact.

Notice how this shows that a product of infinitely many non-compact spaces is never locally compact.

$(X, \mathcal{T}_p)$ **is not compact**.
The open cover $\{\{x, p\} : x \in X\}$ does not admit any finite subcover.

$(X, \mathcal{T}_p)$ **is locally compact and first-countable**.
A basis of neighbourhoods of p consists of the open set $\{p\}$. A basis of neighbourhoods of some other point x consists of the open set $\{x, p\}$. In both cases we have a countable neighbourhood basis and a compact neighbourhood.

$(X, \mathcal{T}_p)$ **is not second-countable**.
As mentioned in the introduction, we are assuming that X is not countable. Then the subspace $X \setminus \{p\}$ is discrete and uncountable, so not second-countable. Since a subspace of a second-countable space is second-countable, X cannot be second-countable.

2.2 Blue squares

$(\mathbb{R}^\omega, \mathcal{T}_{box})$ **is disconnected**.
Let $S \subset \mathbb{R}^\omega$ be the set of sequences converging to 0. We claim that S is open. Indeed, let $\mathbf{x} \in \mathbb{R}^\omega$. Define $U = \prod_{n \geq 1}(x_n - 2^{-n}, x_n + 2^{-n})$. Then U is open in the box topology, and any sequence contained in U will also converge to 0. Thus $\mathbf{x} \in U \subseteq S$, which implies that S is open. The exact same proof shows that $\mathbb{R}^\omega \setminus S$ is open. Since both of these sets are clearly non-empty, $\mathbb{R}^\omega$ is disconnected for the box topology.

This implies that $(\mathbb{R}^\omega, \mathcal{T}_{box})$ is not **path-connected**, **simply-connected** or **contractible**.

$(\mathbb{R}^\omega, \mathcal{T}_{box})$ **is not first-countable**.

We show that $\mathbf{0} = (0, 0, \dots)$ does not have a countable neighbourhood basis. So let $\mathcal{B} = (B_n)_{n\geq 1}$ be a collection of open neighbourhoods of $\mathbf{0}$. We show that $\mathcal{B}$ cannot be a basis by finding an open neighbourhood of $\mathbf{0}$ that does not contain any of the B_n.

For this we use a diagonal argument. For each n, the set $\pi_n(B_n)$ is an open neighbourhood of 0 (recall that, more or less by definition of the product topology, projections are open), so we can find a strictly smaller open neighbourhood $0 \in U_n \subsetneq \pi_n(B_n)$. We then define $U := \prod_{n\geq 1} U_n$, which is open in the box topology. But by construction $\pi_n(B_n) \nsubseteq \pi_n(U)$, so $B_n \nsubseteq U$. We have found the desired open set, and we conclude.

This implies that $(\mathbb{R}^\omega, \mathcal{T}_{box})$ is not **second-countable** or **metrizable**.

$(X, \mathcal{T}_p)$ **is contractible**.
We want to find a map $H : X \times I \to X$ such that $H(x, 0) = x$ and $H(x, 1) = p$ for all $x \in X$. We define $H(x, t) = p$ for all $t > 0$. Then H is continuous. Indeed, let $U \subseteq X$ be an open set, so $p \in U$. Then $H^{-1}(U) = U \times \{0\} \cup X \times (0, 1] = U \times [0, 1] \cup X \times (0, 1]$, which is open in $X \times I$.

This implies that $(X, \mathcal{T}_p)$ is **path-connected** and **simply connected**. It also implies that it is **locally path-connected**, since $\{\{x, p\} : x \in X\}$ is a basis of path-connected open sets. The fact that $\{x, p\}$ is path connected follows from the fact that its subspace topology is still the particular point topology, so it is path-connected.

2.3 Red stars

$(X, \mathcal{T}_{cof})$ **is not path-connected**.
See this post on StackExchange.

Notice that since $(X, \mathcal{T}_{cof})$ is connected, and connected locally path-connected spaces are automatically path-connected, this implies that $(X, \mathcal{T}_{cof})$ is not **locally path-connected**.

S^n **is not contractible**.
You know this already for $n = 1$. For higher dimensions, it is a much harder statement, in fact, it is equivalent to Brower's fixed-point theorem in dimension $(n + 1)$. You have proven Brower's fixed-point theorem in dimension 2, but for higher dimensions you need some more serious algebraic topology. However, looking at the proof of Brower's fixed-point theorem in dimension 2 that you saw, it should not be hard to believe that the following statements are equivalent:

- There exists a continuous map $D^{n+1} \to D^{n+1}$ without fixed points.
- There exists a retraction $D^{n+1} \to S^n$ (i.e., a continuous map that is the identity when restricted to S^n).
- S^n is contractible.

$(\mathbb{R}^\omega, \mathcal{T}_{prod})$ **is (completely) metrizable**.
More generally, a countable product of (complete) metric spaces is (completely) metrizable with the product topology. See the red stars of the construction table for a reference.

$(\mathbb{R}^\omega, \mathcal{T}_{box})$ **is not locally compact**.
See this post on StackExchange.

$(\mathbb{R}^\omega, \mathcal{T}_{box})$ **is not locally path-connected**.

This should be easy to see once you have solved exercise 2(c) in paragraph 25 of Munkres, which involves identifying the components of $(\mathbb{R}^\omega, \mathcal{T}_{box})$. You can find here a solution to that exercise.

$(\mathbb{R}^\omega, \mathcal{T}_{box})$ **is regular**.

It is actually easier to show that it is completely regular: for all C closed and all $x \notin C$, there exists a continuous map $f : \mathbb{R}^\omega \to [0, 1]$ such that $f(x) = 0$ and $f(C) = 1$. It should be clear that completely regular implies regular. For a proof of complete regularity, see this post on StackExchange.

3 Constructions table

3.1 Green circles

A subspace of a locally compact space is not necessarily locally compact.
$\mathbb{R}$ is locally compact but $\mathbb{Q}$ is not.

A closed subspace of a locally compact space is locally compact.
Let X be locally compact, C a closed subset and $x \in C$. Since X is locally compact, there exists an open set U and a compact set K such that $x \in U \subseteq K$. Then $x \in C \cap U \subseteq C \cap K$. The set $C \cap U$ is open in C. The set $C \cap K$ is a closed subset of K, so it is compact. Therefore C is locally compact with the subspace topology.

A closed subspace of a normal space is normal.
Let X be normal, C a closed subset of X. Let F_1, F_2 be two closed subsets of C, with respect to the subspace topology, such that $F_1 \cap F_2 = \emptyset$. Since C is closed in X, the F_i are also closed in X, so there exist open sets U_i such that $F_i \subseteq U_i$ and $U_1 \cap U_2 = \emptyset$. Then $C \cap U_i$ are open in C, $F_i \subseteq C \cap U_i$ and $(C \cap U_1) \cap (C \cap U_2) = \emptyset$. Therefore C is normal with the subspace topology.

A finite product of locally compact spaces is locally compact.
Let $X_1, \ldots, X_n$ be locally compact spaces, and let $\mathbf{x} \in X := \prod_{i=1}^n X_i$. Since each X_i is locally compact, we can find U_i open in X_i and K_i compact such that $x_i \in U_i \subseteq K_i$. Then $U := \prod_{i=1}^n U_i$ is open in X, $K := \prod_{i=1}^n K_i$ is compact, and $\mathbf{x} \in U \subseteq K$. Therefore X is locally compact with the product topology.

A finite product of locally path-connected spaces is path-connected.
Let $X_1, \ldots, X_n$ be locally compact spaces, and for each i let $\mathcal{B}_i$ be a basis of path-connected open sets of X_i. Then $\mathcal{B} = \{\prod_{i=1}^n U_i : U_i \in \mathcal{B}_i\}$ is a basis of $\prod_{i=1}^n X_i$, and each open set of $\mathcal{B}$ is path-connected, since the product of path-connected spaces is path-connected.

A countable product of first- (resp. second-) countable spaces is first- (resp. second-) countable.
Let X_n be second-countable, $\mathcal{B}_n$ a countable basis for X_n. Then $\mathcal{B} := \{\prod_{n\geq 1} B_n : B_n \in (\mathcal{B}_n \cup \{X_n\})\} \cap \mathcal{T}_{prod}$ is a countable basis for $\prod_{n\geq 1} X_n$ with the product topology.

The proof for first-countable spaces is the same.

An uncountable product of second-countable spaces may not even be first-countable.
We have seen in exercise 5.7 that $[0,1]^{[0,1]}$ is compact but not sequentially compact, and in class that first-countable and compact implies sequentially compact, so $[0,1]^{[0,1]}$ cannot be first-countable. However, $[0,1]$ itself is second-countable.

An uncountable product of (complete) metric spaces might not be metrizable.
We have just seen that $[0,1]^{[0,1]}$ is not first-countable, so it is not metrizable.

A quotient of a contractible space may not even be simply connected.
The covering map $\mathbb{R} \to S^1 : x \mapsto e^{2\pi i x}$ is open, so it is a quotient map (the corresponding equivalence relation is $x \sim y \Leftrightarrow (x - y) \in \mathbb{Z}$). But $\mathbb{R}$ is contractible and S^1 is not simply

connected.

This also implies the same for **continuous images**.

A metric space homeomorphic to a complete metric space may not be complete. We have seen that $(0,1) \cong \mathbb{R}$, but $\mathbb{R}$ is complete while $(0,1)$ is not.

Still, at the risk of being pedantic, I want to mention that being *completely metrizable*, meaning that there exists some complete distance inducing the topology, is a property that is stable under homeomorphism. To see what I mean, $(0,1)$ is not complete with its standard distance, but we can identify it with $\mathbb{R}$ and consider instead that distance, which is complete and induces the same topology.

3.2 Blue squares

An infinite product of locally path-connected spaces may not be locally path-connected. More generally, an infinite product of non-path-connected spaces will never be locally path-connected. This is the exact same proof that $(\mathbb{R}^\omega, \mathcal{T}_{prod})$ is not locally compact.

A product of regular spaces is regular.
We will use the following characterization of regularity: a space X is regular if and only if for all $x \in X$ and for any open neighbourhood U of x there exists an open set V such that $x \in V \subseteq \overline{V} \subseteq U$. I proved this in my exercise class, but if you've never heard of it, it is really just a reformulation of the definition. If you want a proof of this, see lemma 31.1 (a) in Munkres.

Let X_i be regular, for i in an arbitrary index set I. Let $\mathbf{x} \in X := \prod_{i\in I} X_i$ and let U be an open neighbourhood of $\mathbf{x}$. Up to shrinking U (check that we can do this without loss of generality), we can assume that U is a basic open set, i.e., $U = \prod_{i\in I} U_i$. Let $J \subseteq I$ be the finite set of indices for which $U_j \neq X_j$. For each $j \in J$, by regularity of X_j we can find an open set V_j such that $x_j \in V_j \subseteq \overline{V_j} \subseteq U_j$. For $i \notin J$, we just let $V_i = X_i$, and we still have $x_i \in V_i \subseteq \overline{V_i} \subseteq U_i$. Moreover, $V := \prod_{i\in I} V_i$ is open in X. Finally: $\mathbf{x} \in V \subseteq \overline{V} = \overline{\prod_{i\in I} V_i} = \prod_{i\in I} \overline{V_i} \subseteq \prod_{i\in I} U_i = U$. Therefore X is regular.

A quotient of a locally compact space might not be locally compact.
Consider the space $\mathbb{R}/\mathbb{Z}$, that is, the quotient space of $\mathbb{R}$ obtained by identifying all integers to a single point, which we denote $* \in \mathbb{R}/\mathbb{Z}$. You can visualize this space as a flower with infinitely many petals, and you should keep it in mind because it is a great source of counterexamples (see also the next paragraph). We claim that $*$ does not have a compact neighbourhood.

Notice that $\mathbb{R}/\mathbb{Z}$ is Hausdorff: this can be proven in an analogous way to Ex. 9.2. So it is enough to show that for any neighbourhood U of $*$, its closure $\overline{U}$ is not compact. Indeed, if U were contained in a compact set K, since K is closed (because $\mathbb{R}/\mathbb{Z}$ is Hausdorff), we would have $\overline{U} \subseteq K$, and so $\overline{U}$ would be compact.

Okay so let U be a neighbourhood of $*$. Up to shrinking U (check that we can do this without loss of generality), we may assume that $U = q(\cup_{n\in\mathbb{Z}}(n-\varepsilon_n, n+\varepsilon_n))$, where q is the quotient map, for real numbers $0 < \varepsilon_n < 1/4$ (the $1/4$ is not important, we just want these intervals not to cover half of $(n, n+1)$). Then it is easy to check that $\overline{U} = q(\cup_{n\in\mathbb{Z}}[n-\varepsilon_n, n+\varepsilon_n])$. Remember that we want to show that $\overline{U}$ is not compact. So here is an open cover with no finite subcover: $\{V_m : m \in \mathbb{Z}\}$, where $V_m := q(\cup_{n\leq m}(n-\varepsilon_n - 1/4, n+\varepsilon_n + 1/4)) \cup U$.

A quotient of a second-countable space might not even be first-countable.
Once again, the counterexample is $\mathbb{R}/\mathbb{Z}$. The proof is very similar to the proof that $(\mathbb{R}^\omega, \mathcal{T}_{box})$ is not first-countable: it involves the same kind of diagonal argument.

This implies the same counterexample for **continuous images**.

The continuous image of a locally path-connected space may not be locally path-connected.
Let S be the topologist's sine curve. Let A, B be its path-connected components. Recall that A and B are locally path-connected while S is not. Then the disjoint union $A \sqcup B$ is locally path-connected, so the natural map $A \sqcup B \to S$ is a continuous surjective map from a locally path-connected space to a non-locally path-connected space.

The closure of a first-countable space may not be first-countable
Consider an uncountable set X with the discrete topology, and $\hat{X}$ its one-point compactification. Then X is first-countable, but its closure $\hat{X}$ is not, because the point at infinity does not have a countable neighbourhood basis. Indeed, by definition of the topology, a neighbourhood of ∞ is a set containing infinity and having finite complement (since a subset of a discrete set is compact if and only if it is finite). So when looking at neighbourhoods of ∞, this works just like the cofinite topology, and you can conclude with the same proof as in Ex. 5.6.

3.3 Red stars

A subspace of a normal space may not be normal.
$\mathbb{R}^\mathbb{R}$ is not normal (see the next paragraph). Once you accept this, here is a counterexample. $[0,1]^\mathbb{R}$ is compact and Hausdorff, so it is normal. But the subspace $(0,1)^\mathbb{R}$ is homeomorphic to $\mathbb{R}^\mathbb{R}$, so it is not normal.

A product of connected spaces is connected.
See this blog post.

A products of normal spaces might not be normal.
The Sorgenfrey line, denoted $\mathbb{R}_\ell$, is the topology on $\mathbb{R}$ described by having the intervals of the form $[a, b)$ as a basis. At the end of paragraph 31 in Munkres, you can find a proof that $\mathbb{R}_\ell$ is normal but $\mathbb{R}_\ell^2$ is not.

If you only care about arbitrary products, then the proof that $\mathbb{R}^\mathbb{R}$ is not normal is, in my opinion, simpler than the one above (and you don't need to work with a new topology). Exercise 9 in paragraph 32 of Munkres gives you a step-by-step approach (but it's still an exercise, not a solution).

A countable product of (complete) metric spaces is (completely) metrizable.
See Theorem 20.5 in Munkres. You will only find a proof of metrizability, but you can prove directly that that same metric is complete, if the spaces you started with are complete.

A quotient of a locally path-connected space is locally path-connected.
This is actually not too hard, once you have a characterization of local path-connectedness that we have not seen in class. This post on StackExchange gives a proof for local connectedness, you

can just add "path-" before every "connected" and the same proof works.

If $(X, \mathcal{T})$ is normal and $\mathcal{T} \subseteq \mathcal{T}'$, then $(X, \mathcal{T}')$ might not even be regular.
The space $\mathbb{R}_K$ is $\mathbb{R}$ endowed with the standard topology to which we add the closed set $K := \{1/n : n \geq 1\}$ (notice that this is not closed in the standard topology, because it clearly contains a sequence converging to 0). At the end of paragraph 31 in Munkres, you can find a proof that $\mathbb{R}_K$ is not regular. However, it has a finer topology that $\mathbb{R}$, which is normal.

If $(X, \mathcal{T})$ is contractible and $\mathcal{T}' \subseteq \mathcal{T}$, then $(X, \mathcal{T}')$ might not even be simply connected.
See this post on StackExchange.

	(X, τ_{tr})	(X, τ_{disc})	(X, τ_{cof})	(X, τ_p)	$\mathbb{R}$	S^1	S^n	$(\mathbb{R}^\omega, \tau_{prod})$	$(\mathbb{R}^\omega, \tau_{box})$
Compact	✓	✗	✓	✗ ∘	✗	✓	✓	✗	✗
Loc. compact	✓	✓	✓	✓ ∘	✓	✓	✓	✗ ∘	✗ *
Connected	✓	✗	✓	✓ •	✓	✓	✓	✓	✗ □
Path-conn.	✓ ∘	✗	✗ *	✓ □	✓	✓	✓	✓	✗ □
Loc. path-conn.	✓ ∘	✓	✗	✓ • □	✓	✓	✓	✓	✗ *
1^{st} count.	✓	✓	✗	✓ ∘	✓	✓	✓	✓	✗ □
2^{nd} count.	✓	✓	✗	✗ ∘	✓	✓	✓	✓	✗ □
T_2	✗	✓	✗	✗ •	✓	✓	✓	✓	✓
T_3	✗	✓	✗	✗ •	✓	✓	✓	✓	✓ *
T_4	✗	✓	✗	✗ •	✓	✓	✓	✓	?
Metrizable	✗	✓	✗	✗ •	✓	✓	✓	✓ *	✗
Complete	✗	✓	✗	✗ •	✓	✓	✓	✓ *	✗
Simply conn.	✓ ∘	✗	✗	✓ □	✓	✗	✓	✓	✗
Contractible	✓ ∘	✗	✗	✓ □	✓	✗	✗ *	✓	✗

	Subspace	Product	Quotient	Image	Homeo	Closure	Finer τ	Coarser τ
Compact	Closed	✓	✓	✓	✓	✗	✗	✓
Loc. compact	Closed ∘	Finite ∘	✗ □	✗	✓	✗	✗	✗
Connected	✗	✓ *	✓	✓	✓	✓	✗	✓
Path-connected	✗	✓	✓	✓	✓	✗	✗	✓
Loc. path-conn.	✗	Finite ∘□	✓ *	✗ □	✓	✗	✗	✗
1st count.	✓	Countable ∘	✗ □	✗	✓	✗ □	✗	✗
2nd count.	✓	Countable ∘	✗ □	✗	✓	✗	✗	✗
T_2	✓	✓	✗	✗	✓	✗	✓	✗
T_3	✓	✓ □	✗	✗	✓	✗	✗ *	✗
T_4	Closed ∘*	✗ *	✗	✗	✓	✗	✗ *	✗
Metrizable	✓	Countable ∘*	✗	✗	✓	✗	✗	✗
Complete	Closed	Countable ∘*	✗	✗	✗ ∘	✗	✗	✗
Simply conn.	✗	✓	✗ ∘	✗	✓	✗	✗	✗ *
Contractible	✗	✓	✗ ∘	✗	✓	✗	✗	✗ *